2020 College Bowl Grades 11–12 Round 4

Question

What is the value of $\sin \frac{23\pi}{4}$?

Answer $-\frac{\sqrt{2}}{2}$ (or equivalent, such as $-\frac{1}{\sqrt{2}}$).

Follow-up

What is the value of $\sin \frac{\pi}{12}$?

Answer $\frac{-1+\sqrt{3}}{2\sqrt{2}}$ (or any equivalent answer, such as $\frac{\sqrt{6}-\sqrt{2}}{4}$).

Question

What is the value of $\sin^2 x + \cos^2 x + \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{6}$?

Answer

2.

Follow-up

If $\sin x + \cos x = \frac{1}{2}$, then what is the value of $\sin^3 x + \cos^3 x$?

Answer

 $\frac{11}{16}$.

Question

At a meeting of four people, every possible pair shook hands once. How many handshakes were there all together?

Answer

Six.

Follow-up

At a party, every possible pair of people shook hands once. How many people attended the party if there were 66 handshakes all together?

Answer

12.

Question

When it is one o'clock, what is the degree measure between the hour and minute hands of a clock?

Answer

30 degrees.

Follow-up

Suppose that the angle between the hour and minute hands of a clock is 60 degrees. If the minute hand is 15 inches long and the hour hand is 10 inches long, what is the distance between the ends of these hands?

Answer

 $5\sqrt{7}$ inches (or equivalent, such as $\sqrt{175}$ inches).